Analysis of Pressure Fluctuations in a Gas-Solid Fluidized Bed

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Periodic fluctuations of pressure can be observed in a normally operated gas-solid fluidized bed, and the dominant frequency of such fluctuations has been studied by several investigators (Hiby, 1967; Lirag and Littman, 1971; Baird and Klein, 1973; Verloop and Heertjes, 1974; Goossens, 1975; Canada et al., 1978; Fan et al., 1981). This note proposes a new dynamic model, which is based on the assumption that fluctuations of the bed height are affected by both the present and time-delayed fluctuations of the gas flow rate through the distributor. According to the present model, the dominant frequency of pressure fluctuations in a fluidized bed corresponds to the marginal frequency of bed instability.

DYNAMIC MODELING OF A GAS-SOLID FLUIDIZED BED

The schematic diagram of the fluidized-bed system is shown in Figure 1 with relevant physical parameters indicated. The x-coordinate is the distance upward from the distributor plate.

The following assumptions are made to derive the dynamic equations of the bed.

(i) The rising velocity of a bubble, U_b, is given by

$$U_b = U_{br} + (U_o - U_{mf}) \tag{1}$$

where U_{br} is the rising velocity of the bubble at the minimum fluidizing condition

(ii) The bubble residence time, T, is implicitly related to the rising velocity of a bubble as

$$L = \int_{t-T}^{t} U_b(t')dt' \tag{2}$$

where L is the height of the bed at any time t.

(iii) The bubble holdup, G, can be obtained from the bubble residence time and the gas flow rate through the distributor as follows:

$$G = A \int_{t-T}^{t} \{U_o(t') - U_{mf}\}dt' \tag{3}$$

Based on the above assumptions, the total mass of the solid particles can be expressed as

$$M_s = (LA - G)\rho_{mf} = \text{const.} \tag{4}$$

It follows that

$$\int_{t-T}^{t} U_{br}(t')dt' = \frac{M_s}{\rho_{mf}A} \tag{5}$$

where it is assumed that the bulk density of the dense phase of the bed is equal to that at the minimum fluidizing condition.

The momentum balance equation of the bed, as derived by Pigford and Baron (1965), is

$$\rho_s(1-\epsilon)\frac{\vec{D}_s\vec{U}_s}{Dt} + \rho_f\epsilon\frac{\vec{D}_f\vec{U}_f}{Dt} = -\vec{\nabla}P + \{\rho_s(1-\epsilon) + \rho_f\epsilon\}\vec{g} \tag{6}$$

If we consider only the mean motion along the bed height and assume that the inertia force due to the fluid motion is negligible, Eq. 6 can be rewritten as

$$\rho_s(1-\epsilon)\frac{D_sU_s}{Dt} = -\frac{\partial P}{\partial x} - \rho_s(1-\epsilon)g \tag{7}$$

Upon integrating this equation with respect to x from 0 to L and expressing the resultant lefthand side of the equation in terms of the center of mass, $\hat{L}$, we have

$$\left(\frac{M_s}{A}\right)\frac{d^2\hat{L}}{dt^2} = (P_B - P_\infty) - \left(\frac{M_s}{A}\right)g \tag{8}$$

With the assumption that the void fraction, ϵ, varies with time but not with position, we can write

$$L = 2\hat{L} \tag{9}$$

Introducing Eq. 9 into Eq. 8, we have

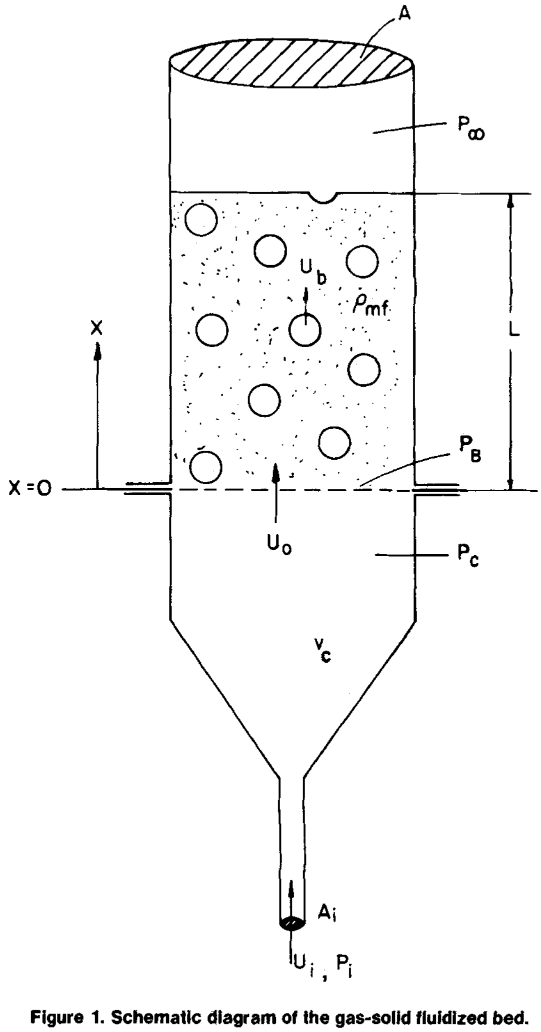

Figure 1. Schematic diagram of the gas-solid fluidized bed.

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$$\left(\frac{M_s}{2A}\right)\frac{d^2L}{dt^2} = (P_B - P_\infty) - \left(\frac{M_s}{A}\right)g \qquad (10)$$

then, with

$$\overline{P}_B - P_\infty = \left(\frac{M_s}{A}\right)g \qquad (11)$$

we obtain

$$\left(\frac{M_s}{2A}\right)\frac{d^2L}{dt^2} = P_B - \overline{P}_B \qquad (12)$$

Combination of Eqs. 1, 2, 5 and 12 gives the relationship between the superficial gas velocity at the distributor plate, $U_o(t)$, and the pressure at the bottom of the bed, $P_B(t)$. Under the assumption that the fluctuations of gas velocity and bubble residence time are sufficiently small compared with the time average ones, the resultant relationship simplifies to

$$\left(\frac{M_s}{2A}\right)\left\{\frac{dU_o(t)}{dt} - \frac{dU_o(t-\overline{T})}{dt} + (\overline{U}_o - U_{mf})\frac{d^2T(t)}{dt^2}\right\} = P_B(t) - \overline{P}_B \qquad (13)$$

The pressure at the bottom of the bed, $P_B(t)$, and that in the plenum, $P_c(t)$, can be related to the gas velocity through the distributor, $U_o(t)$, as follows (Moritomi et al., 1980):

$$(i) \quad P_c(t) - P_B(t) = K_D\, U_o^n(t) \qquad (14)$$

and

$$(ii) \quad v_c\frac{dP_c(t)}{dt} = (P_iU_i)A_i - \overline{P}_cAU_o(t) \qquad (15)$$

From Eqs. 13, 14 and 15, we obtain

$$v_c\left(\frac{M_s}{2A}\right)\left\{\frac{d^2U_o(t)}{dt^2} - \frac{d^2U_o(t-\overline{T})}{dt^2}\right\} + v_cnU_0^{n-1}(t)K_D\frac{dU_o(t)}{dt} + \overline{P}_cAU_o(t) = -(\overline{U}_o - U_{mf})v_c\left(\frac{M_s}{2A}\right)\frac{d^3T(t)}{dt^3} + (P_iU_i)A_i \qquad (16)$$

This equation characterizes the dynamics of a gas-solid fluidized bed in terms of pertinent parameters including both the internal noise in the bed and the external noise in the gas supply line. Equation 16 indicates that the residence time fluctuations induced by the deformation and coalescence of gas bubbles can initiate the oscillation of the fluidized bed even when the external noise in the gas supply line is absent.

Transfer Function

The superficial gas velocity at the distributor plate, $U_o(t)$, and the bubble residence time, $T(t)$, can be decomposed into its time average and fluctuating components, respectively, i.e.,

$$\left.\begin{aligned} U_o(t) &= \overline{U}_o + U_o'(t) \\ T(t) &= \overline{T} + T'(t) \end{aligned}\right\} \qquad (17)$$

Introducing Eq. 17 into Eq. 16 and 13 and assuming that no external noise exists in the gas supply line, we obtain, respectively,

$$\left\{\frac{d^2U_o'(t)}{dt^2} - \frac{d^2U_o'(t-\overline{T})}{dt^2}\right\} + \left(\frac{2n\overline{U}_o^{n-1}K_DA}{M_s}\right)\frac{dU_o'(t)}{dt} + \left(\frac{2\overline{P}_cA^2}{v_cM_s}\right)U_o'(t) = -(\overline{U}_o - U_{mf})\frac{d^3T'(t)}{dt^3} \qquad (18)$$

and

$$\left(\frac{M_s}{2A}\right)\left\{\frac{dU_o'(t)}{dt} - \frac{dU_o'(t-\overline{T})}{dt} + (\overline{U}_o - U_{mf})\frac{d^2T'(t)}{dt^2}\right\} = P_B'(t) \qquad (19)$$

for the fluctuating component.

Taking the Laplace transform of Eqs. 18 and 19, we have, respectively,

$$\left\{s^2(1-e^{-s\overline{T}}) + \left(\frac{2n\overline{U}_o^{n-1}K_DA}{M_s}\right)s + \left(\frac{2\overline{P}_cA^2}{v_cM_s}\right)\right\}L\{U_o'(t)\} = -(\overline{U}_o - U_{mf})s^3L\{T'(t)\} \qquad (20)$$

and

$$s(1-e^{-s\overline{T}})L\{U_o'(t)\} + (\overline{U}_o - U_{mf})s^2L\{T'(t)\} = \left(\frac{2A}{M_s}\right)L\{P_B'(t)\} \qquad (21)$$

Eliminating $L\{U_o'(t)\}$ from Eqs. 20 and 21, we obtain the transfer function, $G(s)$, relating $P_B'(t)$ and $T'(t)$, as

$$L\{P_B'(t)\} = \left(\frac{M_s}{2A}\right)(\overline{U}_o - U_{mf})G(s)L\{T'(t)\} \qquad (22)$$

where

$$G(s) = \frac{\left(\frac{2n\overline{U}_o^{n-1}K_DA}{M_s}\right)s^3 + \left(\frac{2\overline{P}_cA^2}{v_cM_s}\right)s^2}{s^2(1-e^{-s\overline{T}}) + \left(\frac{2n\overline{U}_o^{n-1}K_DA}{M_s}\right)s + \left(\frac{2\overline{P}_cA^2}{v_cM_s}\right)} \qquad (23)$$

Equation 22 indicates that the characteristics of the pressure fluctuations at the bottom of the bed can be inferred from those of the bubble residence time. For example, the intensity of pressure fluctuations at the bottom of the bed

$$\sqrt{\overline{(P_B')^2}}$$

can be obtained by considering the Fourier transform version of Eq. 22 and then by taking the inverse Fourier transform of the resultant power spectrum, i.e.,

$$\sqrt{\overline{(P_B')^2}} = \left(\frac{M_s}{2A}\right)(\overline{U}_o - U_{mf})\frac{1}{\sqrt{2\pi}} \times \left[\int_{-\infty}^{\infty} G(j\omega)G^*(j\omega)F\{T'(t)\}F^*\{T'(t)\}d\omega\right]^{1/2} \qquad (24)$$

where $F\{T'(t)\}$ is the Fourier transform of the residence time fluctuations and * denotes its complex conjugate. Equation 24 shows that the intensity of the pressure fluctuations induced by the fluctuations of bubble residence time is proportional to

$$\left(\frac{M_s}{2A}\right)\cdot(\overline{U}_o - U_{mf})$$

and depends on the product

$$G(j\omega)\cdot F\{T'(t)\}$$

Fluidized Bed Instability

For the special case of $T'(t) = 0$, Eq. 20 reduces to

$$\{\theta^2(1-e^{-\theta}) + \alpha\theta + \beta\}L\{U_o'(t)\} = 0 \qquad (25)$$

where

$$\left.\begin{aligned} \theta &= \omega\overline{T} \\ \alpha &= \frac{2n\overline{U}_o^{n-1}K_DA\overline{T}}{M_s} \\ \beta &= \frac{2\overline{P}_cA^2\overline{T}^2}{v_cM_s} \end{aligned}\right\} \qquad (26)$$

The fluidized bed is unstable if the roots of Eq. 25 for s have positive real parts. In other words, the bed instability depends on the magnitudes of two parameters, α and β, in Eq. 25. In what follows only the frequency at the threshold of instability or the frequency, which induces an infinite gain in Eq. 23, is discussed.

Let us set $s = j\omega$ in Eq. 25, at which the bed oscillates with the frequency of $f = \omega/2\pi$. Equating separately the real and imaginary parts of Eq. 25 to zero yields, respectively,

$$-\omega^2(1 - \cos\omega\overline{T}) + \left(\frac{2\overline{P}_c A^2}{v_c M_s}\right) = 0 \tag{27}$$

$$-\omega \sin\omega T + \left(\frac{2n\overline{U}_o^{n-1} K_D A}{M_s}\right) = 0 \tag{28}$$

Examination of Eq. 28 shows that a positive real root of ω that is physically realizable exists only when

$$\frac{2m\pi}{\overline{T}} < \omega < \frac{(2m+1)}{\overline{T}}, \qquad m = 0,1,2,\ldots \tag{29}$$

while no such constraints need to be imposed on the roots of Eq. 27.

All the parameters in both Eqs. 27 and 28 are fixed for a given operating condition. Therefore, solving these equations for K_D, we see that Eq. 25 holds only when

$$K_D = \left(\frac{M_s}{n\overline{U}_o^{n-1}A}\right)\sqrt{\left(\frac{-2\overline{P}_c A^2}{v_c M_s \omega_k^2}\right)\left\{1 - \left(\frac{\overline{P}_c A^2}{v_c M_s \omega_k^2}\right)\right\}}, \qquad k = 1,2,3,\ldots \tag{30}$$

where ω_k is the kth root of Eq. 27.

Observation of the motion of solid particles has indicated that a portion of the solid particles at the bottom of the bed tends to accumulate on the distributor and block the holes on it (see, e.g., Briens et al., 1980). This means that the value of K_D in Eq. 28 is not fixed but varies appreciably during fluidization. We shall, however, assume that K_D does not vary arbitrarily but varies according to Eq. 30. Then, Eq. 27, subject to the constraints of Eq. 29, gives rise to the frequency at the threshold of instability of the bed, which induces the infinite gain in Eq. 23.

Equation 30 shows that angular frequency, ω_1, corresponding to the limiting condition of $K_D = 0$, is inversely proportional to the square root of L_{mf}, i.e.,

$$\omega_1 = 2\pi f_1 = \left(\frac{\overline{P}_c A}{v_c M_s}\right)^{1/2} = \left(\frac{\overline{P}_c A}{v_c \rho_{mf}}\right)^{1/2} \frac{1}{\sqrt{L_{mf}}} \tag{31}$$

This relation implies that the kinetic energy of particles and the pressure work at the bottom of the bed are in equilibrium. Equation 31 is essentially identical to that of Moritomi et al. (1980) and similar to that of Wong and Baird (1971).

DISCUSSION

Experimentally Determined Amplitude of Pressure Fluctuations

The amplitudes of pressure fluctuations in the gas-solid fluidized bed were measured by several investigators, e.g., Lirag and Littman (1971) and Fan et al. (1981). Their experimental results are plotted in Figure 2 against a new parameter, $(M_s/2A)(\overline{U}_o - U_{mf})$, derived in the present work (Eq. 24). Notice that the measured amplitudes in these experiments are roughly proportional to this parameter independent of the size of solid particles.

Experimentally Determined Frequency of Oscillation

Equation 27 indicates that the frequency of oscillation depends on the mean bubble residence time, $\overline{T}$, the average plenum pressure, $\overline{P}_c$, the plenum volume, v_c, the column cross-sectional area, A, and the bed mass, M_s. These parameters, except the mean bubble residence time, are directly measurable without difficulty. The mean bubble residence time cannot be easily determined, because the average rising velocity of a bubble at the minimum fluidizing condition, $\overline{U}_{br}$, cannot be directly measured.

Assume that $\overline{U}_{br}$ may be estimated from (Davidson and Harrison, 1963)

$$\overline{U}_{bm} = 0.71\sqrt{gD_{em}} \tag{32}$$

Further assume that the maximum gas bubble diameter, D_{em}, is given by (Mori and Wen, 1975)

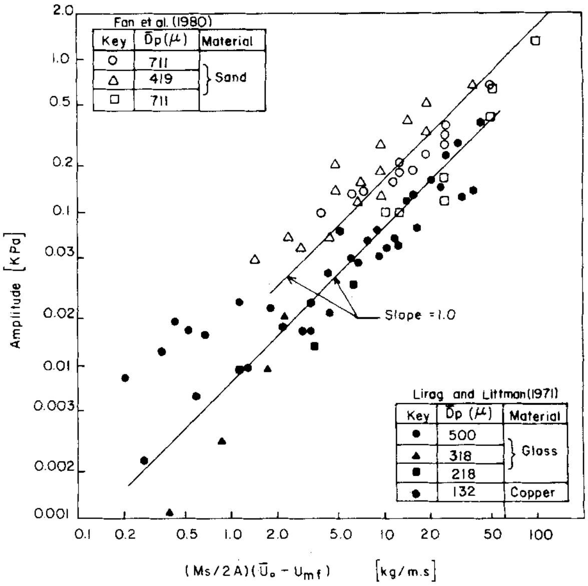

Figure 2. Experimentally determined amplitudes vs. a new parameter $(M_s/2A)(\overline{U}_o - U_{mf})$.

$$D_{em} = 0.652\{A(\overline{U}_o - U_{mf})\}^{2/5} \tag{33}$$

Then, the mean bubble residence time, $\overline{T}$, can be estimated from Eq. 5 as

$$\overline{T} = \frac{M_s}{\rho_{mf} A \overline{U}_{br}} = \frac{L_{mf}}{\overline{U}_{bm}} \tag{34}$$

To compare the proposed model with experimental data on fluidized-bed fluctuations, the dimensionless parameters, θ and ξ, are defined as

$$\theta = \omega\overline{T} = 2\pi f \frac{L_{mf}}{\overline{U}_{br}}$$

$$\xi^2 = \frac{2\overline{P}_c A^2}{v_c M_s}\overline{T}^2 = \frac{2\overline{P}_c L_{mf}}{(v_c/A)\rho_{mf}\overline{U}_{br}^2} \tag{35}$$

Then, Eqs. 27 and 29 can be rewritten, respectively, as

$$\theta^2(1 - \cos\theta) - \xi^2 = 0 \tag{36}$$

$$2m\pi < \theta < (2m+1)\pi, \qquad m = 0,1,2,\ldots \tag{37}$$

This set of relationships between θ and ξ is indicated by a series of discontinuous solid lines in Figure 3. The experimental data of the dominant frequencies obtained by several investigators are plotted in the same figure. One or more of the parameters needed in this plot, for example, the plenum volume, v_c, and the ratio of the gas velocity to the minimum fluidizing velocity, $\overline{U}_o/U_{mf}$, were not available. Therefore, they have been estimated approximately.

Most of the data in Figure 3 appear to satisfy Eq. 36 reasonably well. Verloop and Heertjes (1974) have shown that the dominant frequency does not monotonously decrease with an increase in the bed height (Figure 2 of their work). A careful examination reveals that the dominant frequency appears to increase or jump abruptly at some bed height. This abrupt jump in the dominant frequency can be explained by the discontinuity in Eq. 36. Goossens (1975) has pointed out that the difference in particle sizes also affects the dependency of frequency on the bed height; it is reasonable to assume that large particles may satisfy Eq. 36 in the range of $0 < \theta < \pi$ while small particles do in the range of $2\pi < \theta < 3\pi$. Figure 3 also includes the experimental results of Wong and Baird (1971) for the pulsed fluidized beds. In their experiments, however, $\overline{U}_o$ for such a bed is sometimes less than U_{mf}. Such cases are excluded. For small particles, the data by Wong and Baird (1971) seem to satisfy Eq. 36. The correlations based on their piston model and those by Moritomi et al. (1980) are also shown in Figure 3. Notice that, while the piston model can roughly estimate the dominant frequency, it cannot account for the occurrence of the apparent

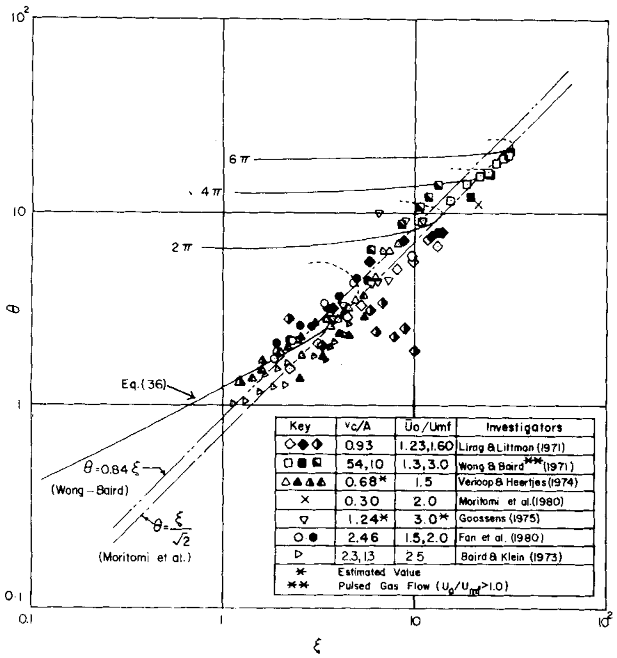

Figure 3. Plot of θ against ξ according to Eqs. 36 and 37.

abrupt jump in the dominant frequency. If we consider that the pressure fluctuations in the bed are induced by the bubble residence time fluctuations, Figure 2, the dominant frequency of the pressure fluctuations can be interpreted as the frequency which induces the infinite gain in Eq. 23. Figure 3 appears to confirm this observation.

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NOTATION

A = column cross section, m^2
A_i = gas inlet pipe cross section, m^2
D_{em} = maximum bubble diameter, m
$\overline{D}_p$ = average particle diameter, m
f = dominant frequency, s^{-1}
G = bubble holdup, m^3
g = gravitational acceleration, m/s^2
K_D = pressure resistance coefficient of the distributor, $N/(m/s)^n \cdot m^2$
L = bed height, m
$\hat{L}$ = height of the center of mass, m
L_{mf} = bed height at the minimum fluidizing condition, m
M_s = total particle mass, kg
n = constant
P = pressure, N/m^2
P_B = pressure at the bottom of the bed, N/m^2
P_c = plenum pressure, N/m^2
P_i = gas inlet pressure, N/m^2
P_∞ = ambient pressure, N/m^2
s = complex variable for Laplace transformation, s^{-1}
t = time
T = bubble residence time, s
U_b = rising velocity of bubble, m/s
U_{bm} = maximum rising velocity of bubble, m/s
U_{br} = rising velocity of bubble at the minimum fluidizing condition, m/s
U_f = linear velocity of fluid, m/s
U_i = inlet gas velocity, m/s
U_s = linear velocity of solids, m/s
U_o = superficial gas velocity through the distributor, m/s
U_{mf} = minimum fluidizing velocity, m/s
v_c = plenum volume, m^3
x = upward distance from the distributor, m
ϵ = void fraction, dimensionless

Greek Letters

α,β,θ = dimensionless variables defined by Eq. 26
ξ = dimensionless variable defined by Eq. 35
ρ_{mf} = bed density at the minimum fluidizing condition, kg/m^3
ρ_f = fluid density, kg/m^3
ρ_s = density of solids, kg/m^3
ω = circular frequency ($=2\pi f$), s^{-1}

Superscripts

— = time average
= fluctuating component

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